

Genuine multipartite system-environment correlations in decoherent dynamics

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We propose relative entropy-based quantifiers for genuine multipartite total, quantum, and classical correlations. These correlation measures are applied to investigate the generation of genuine multiparticle correlations in decoherent dynamics induced by the interaction of two qubits with local-independent environments. We consider amplitude- and phase-damping channels and compare their capabilities to spread information through the creation of many-body correlations. We identify changes in behavior for the genuine 4- and 3-partite total correlations and show that, contrary to amplitude environments, phase-noise channels transform the bipartite correlation initially shared between the qubits into genuine multiparticle system-environment correlations.

I. INTRODUCTION

The nonlocality [1, 2], nonseparability [3, 4], and quantumness [5, 6] of correlations in composite systems are currently recognized as important ingredients for the efficiency of protocols in quantum information science. Substantial progress has been achieved latterly regarding the characterization, identification, and quantification of total, classical, and quantum correlations [7–21]. The non-classicality originated from local indistinguishability of quantum states is the more recent paradigm for analyzing correlations. This kind of correlation was studied mostly in the case of bipartite systems, with novel and interesting results already obtained. By its turn, the investigation of (genuine) multipartite quantum and classical correlations has been receiving its deserved attention only very recently [22–43]. In this article we are interested in this last scenario. We shall define measures for genuine n -partite correlations (Section II) and study how these correlations develop during decoherent system-environment dynamics (Section III).

The thorough investigation of the decoherence process [44–46] is an important issue to be addressed towards large scale implementations of, for instance, quantum computers, quantum simulators, and communication protocols. The decoherence phenomenon is a result of the inevitable interaction between a quantum system and its surroundings. Let us consider a system s and its environment E in an initial product state $\rho_{sE} = \rho_s \otimes |0_E\rangle\langle 0_E|$. The dynamics of the whole closed system is unitary, i.e., $\rho_{sE}(t) = U_{sE}(t)\rho_{sE}U_{sE}^\dagger(t)$ with $U_{sE}(t)U_{sE}^\dagger(t) = \mathbb{I}$, where \mathbb{I} is the identity operator. Then, by tracing out the environmental variables we write down the system's evolved state in the Kraus or operator-sum representation: $\mathcal{E}(\rho_s) = \sum_i K_i \rho_s K_i^\dagger$, where $K_i := \langle i_E | U_{sE}(t) | 0_E \rangle$ are linear operators on the state space of the system, \mathcal{H}_s , such that $\sum_i K_i^\dagger K_i \leq \mathbb{I}$. The quantum operation \mathcal{E} can be used to describe general transformations between quantum states [47, 48]. One

can verify that the dynamical map for the evolution of the system-environment state:

$$U_{sE}|\psi_s\rangle \otimes |0_E\rangle = \sum_i K_i |\psi_s\rangle \otimes |i_E\rangle, \quad (1)$$

leads to the same motion equation for the system state as shown above. Thus, we can obtain the Kraus' operators describing the noise channel acting on the system using a phenomenological approach [49, 50] or via quantum process tomography [51, 52] and use this information to investigate, for example, the system-environment correlations, without worry about the usually complicated structure of the environment.

A possible, partial, classification of multipartite quantum states concerning its correlations, or with regard to the operations needed to generate such correlations, can be introduced as follows. For a n -partite system prepared in a product state vector $|\psi_{\text{init}}\rangle = |\psi_{01} \cdots \psi_{0n}\rangle$, any uncorrelated n -partite product state of this system,

$$\rho_{1\dots n}^p = \rho_1 \otimes \cdots \otimes \rho_n, \quad (2)$$

can be created by means of local quantum operations (LQO). The sub-index is in $|\psi_{\text{init}}\rangle$ specifies the state and subsystem, respectively, and we use throughout this article the notation $|\psi_{01} \cdots \psi_{0n}\rangle := |\psi_{01}\rangle \otimes \cdots \otimes |\psi_{0n}\rangle$. Starting with the system in state $|\psi_{\text{init}}\rangle$, any n -partite classically-correlated state can be prepared via local classical operations (LCO) coordinated by the exchange of classical communication (CC) and has the form:

$$\rho_{1\dots n}^c = \sum_{i1, \dots, in} p_{i1, \dots, in} |\psi_{i1} \cdots \psi_{in}\rangle \langle \psi_{i1} \cdots \psi_{in}|, \quad (3)$$

where the states $\{|\psi_{is}\rangle\} \in \mathcal{H}_s$ form a complete ($\sum_{is} |\psi_{is}\rangle \langle \psi_{is}| = \mathbb{I}_s$) orthonormal ($\langle \psi_{is} | \psi_{js} \rangle = \delta_{ij}$), and therefore distinguishable, basis for the subsystem s . Above, by LCO we mean (complete) transformations between the pointer basis states [44]. It can be seen that if the probability distribution in the state $\rho_{1\dots n}^c$ factorizes, i.e., $p_{i1, \dots, in} = p_{i1} \cdots p_{in}$, there is no correlation at all in the system, i.e., it is a n -partite product state. A n -partite separable but quantumly correlated state needs LQO and CC to be generated from $|\psi_{\text{init}}\rangle$. This kind of state has the general form

$$\rho_{1\dots n}^s = \sum_i p_i \rho_{i1} \otimes \cdots \otimes \rho_{in}, \quad (4)$$

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with $\{p_i\}$ being a joint probability distribution and $\{\rho_{is}\}$ being noncommuting density operators. We note that if the density operators ρ_{is} commute for different i , then the state $\rho_{1\dots n}^s$ is a n -partite classically-correlated state. If in addition the probability distribution $\{p_i\}$ factorizes, then $\rho_{1\dots n}^s$ is a n -partite product state. The N -partite entangled, or non-separable, states $\rho_{1\dots n}^e$ cannot be prepared locally, requiring direct or mediated interaction for its generation. One famous example of entangled state is the GHZ state $2^{-1/2}(|0_1 \dots 0_n\rangle + |1_1 \dots 1_n\rangle)$ [53, 54], where $\{|0_s\rangle, |1_s\rangle\}$ is the one-qubit computational basis.

With the aim of quantifying the different kinds of correlation in an unified manner, Modi *et al.* [41] introduced measures of correlation using the relative entropy [55],

$$S(\rho||\sigma) = \text{tr}(\rho(\log_2 \rho - \log_2 \sigma)), \quad (5)$$

to estimate the “distance” between two states ρ and σ . The total correlation in a n -partite state $\rho_{1\dots n}$ is quantified by how distinguishable or how distant it is from an uncorrelated n -partite product state, i.e.,

$$\begin{aligned} I(\rho_{1\dots n}) &= \min_{\rho_{1\dots n}^p} S(\rho_{1\dots n}||\rho_{1\dots n}^p) \\ &= S(\rho_{1\dots n}||\rho_1 \otimes \dots \otimes \rho_n). \end{aligned} \quad (6)$$

The last equality was established in Ref. [41] and shows that the closest n -partite product state of any state $\rho_{1\dots n}$ is obtained from its marginal states in the product form. Recalling the state classification presented in the previous paragraph, the quantum part of the correlation in $\rho_{1\dots n}$ can be defined as its minimal distance from n -partite classically-correlated states:

$$\begin{aligned} Q(\rho_{1\dots n}) &= \min_{\rho_{1\dots n}^c} S(\rho_{1\dots n}||\rho_{1\dots n}^c) \\ &= S(\rho_{1\dots n}||\chi_{1\dots n}^p), \end{aligned} \quad (7)$$

with $\chi_{1\dots n}^p = \sum_{i_1, \dots, i_n} p_{i_1, \dots, i_n} |\psi_{i_1} \dots \psi_{i_n}\rangle \langle \psi_{i_1} \dots \psi_{i_n}|$ and $p_{i_1, \dots, i_n} = \langle \psi_{i_1} \dots \psi_{i_n} | \rho_{1\dots n} | \psi_{i_1} \dots \psi_{i_n} \rangle$ [41]. In the second equality for Q , we leave implicit the minimization over local basis needed to find the closest classically-correlated state $\chi_{1\dots n}^p$. Finally, the classical part of the correlations in $\rho_{1\dots n}$ is given by the total correlation of $\chi_{1\dots n}^p$:

$$C(\rho_{1\dots n}) = \min_{\rho_{1\dots n}^p} S(\chi_{1\dots n}^p||\rho_{1\dots n}^p). \quad (8)$$

Though the state classification and the correlation quantifiers presented above are very useful in several contexts, they are restricted in a certain sense because they classify the system state without considering the possibility of a more complex distribution for the correlations. For instance, in studying multipartite spin systems, which may have a far more intricate density operator with groups or cluster of spins in different families of states, it would be desirable to generalize the results presented in the last two paragraphs in order to include and quantify such complexity. As we will discuss in the

sequence, the concept of genuine multipartite correlation fits well for the study of these more general scenarios and can be utilized as the starting point to define direct generalizations of the correlation quantifiers discussed above. Addressing this subject, Bennett *et al.* (see Ref.[43]) postulated that if a n -partite state has no genuine n -partite correlation, then we cannot create genuine n -partite correlation by adding a subsystem in a product state or by performing trace non-increasing local quantum operations. Also, we cannot create genuine $(n+1)$ -partite correlation by splitting a subsystem in two. They proved that the following definitions satisfy these requirements. A n -partite state has genuine n -partite correlation only if it is non-product under any bipartite cut of the system. For $k \leq n$, a n -partite state has genuine k -partite correlation only if there exists a subset of k subsystems presenting genuine k -partite correlation. Bennett and collaborators also defined the degree of correlation of a n -partite state as the maximum number k of subsystems possessing genuine k -partite correlation.

II. QUANTIFIERS FOR GENUINE MULTIPARTITE CORRELATIONS

From the previous definition, it follows that a n -partite state has genuine n -partite correlation only if it is non-product under any bipartite cut of the system, i.e., if $\rho_{1\dots n} \neq \rho_{c_1} \otimes \rho_{c_2}$, where c_1 (c_2) indicates a group with a number n_1 (n_2) of subsystems, and $n_1 + n_2 = n$. That is to say, if $\rho_{1\dots n}$ possesses genuine n -partite correlation then there does not exist two completely uncorrelated clusters of particles in the system. Thus, a measure for genuine n -partite total correlation of a n -partite state can be defined as

$$I_n(\rho_{1\dots n}) := \min_{(c_1, c_2)} S(\rho_{1\dots n}||\rho_{c_1} \otimes \rho_{c_2}), \quad (9)$$

where the minimum is taken over all possible bipartitions (c_1, c_2) of the system. Therefore the quantity I_n measures the minimum distance between $\rho_{1\dots n}$ and states that do not possess genuine n -partite correlation. The quantifier for genuine n -partite total correlation $I_n(\rho_{1\dots n})$ was noticed by Giorgi and colleagues in Ref. [42]. They also defined measures for genuine classical and quantum correlations using a discord-like approach, i.e., assuming additivity of mutual information in classical and quantum correlations. In this article we start from a direct extension of the operational classification of states discussed in Section I and use relative entropy-based distinguishability measures to define correlation quantifiers. Regarding total correlations, we can also look to the structure of the system’s correlation by defining a measure for genuine k -partite total correlation of a n -partite state:

$$I_k(\rho_{1\dots n}) := \max_{\rho_k} I_k(\rho_k). \quad (10)$$

In the last equation, the maximum is taken over all states comprising k subsystems and aims to identify the most correlated k -partite group of particles in the system. For example, if $k = n - 1$, $I_{n-1}(\rho_{1\dots n})$ is the maximal genuine $(n - 1)$ -partite total correlation of the states obtained by tracing out one subsystem of $\rho_{1\dots n}$. This last quantifier, $I_k(\rho_{1\dots n})$, can be applied to study the degree of correlation of a system and can find application, for instance, in the study of quantum phase transitions in critical systems [56].

Based on the discussion about classically-correlated states and quantum correlation in non-classical states, presented in the introductory section, we propose the following definition. A n -partite state $\rho_{1\dots n}$ has genuine n -partite quantum correlation only if it is not a classical state under any bipartite cut of the system, viz., $\rho_{1\dots n} \neq \chi_{c_1 c_2}^\rho$, with $\chi_{c_1 c_2}^\rho = \sum_{i c_1, i c_2} p_{i c_1, i c_2} |\psi_{i c_1} \psi_{i c_2}\rangle \langle \psi_{i c_1} \psi_{i c_2}|$, where $\{p_{i c_1, i c_2} = \langle \psi_{i c_1} \psi_{i c_2} | \rho_{1\dots n} | \psi_{i c_1} \psi_{i c_2} \rangle\}$ is a probability distribution and $\{|\psi_{i c_s}\rangle\}$ is an orthonormal base for \mathcal{H}_{c_s} . From this definition, a quantifier of genuine n -partite quantum correlation of a n -partite state follows as

$$Q_n(\rho_{1\dots n}) := \min_{(c_1, c_2)} S(\rho_{1\dots n} || \chi_{c_1 c_2}^\rho). \quad (11)$$

By definition, $Q_n(\rho_{1\dots n}) \geq 0$ with equality only in cases where $\rho_{1\dots n} = \chi_{c_1 c_2}^\rho$. A measure for genuine k -partite quantum correlation of a n -partite system is defined in an analogous manner to the case of total correlation:

$$Q_k(\rho_{1\dots n}) := \max_{\rho_k} Q_k(\rho_k). \quad (12)$$

One possible definition for genuine n -partite classical correlation of a n -partite state is given as follows. A n -partite state $\rho_{1\dots n}$ possesses genuine n -partite classical correlation only if its closest n -partite classical state $\chi_{1\dots n}^\rho$ has genuine n -partite total correlation, namely, if the classical probability distribution p_{i_1, \dots, i_n} does not factorizes under any bipartite cut of $\chi_{1\dots n}^\rho$: $p_{i_1, \dots, i_n} \neq p_{c_1} p_{c_2}$. Based on this definition, we propose the following quantifiers for genuine n -partite classical correlation:

$$C_n(\rho_{1\dots n}) := I_n(\chi_{1\dots n}^\rho), \quad (13)$$

and for genuine k -partite classical correlation:

$$C_k(\rho_{1\dots n}) = \max_{\chi_k^\rho} I_k(\chi_k^\rho), \quad (14)$$

of a n -partite state. The states χ_k^ρ in the last equation are obtained by tracing out all but the chosen k subsystems of $\chi_{1\dots n}^\rho$ and the maximization is made over all possibilities for χ_k^ρ . In the next section we will apply some of the quantifiers of genuine n -partite correlations introduced here to study the generation of genuine multipartite correlations in decoherent dynamics.

III. SYSTEM-ENVIRONMENT CORRELATIONS IN DECOHERENT DYNAMICS

Let us consider a two-qubit system initially prepared in a Werner's state

$$\rho_{ab}^w = (1 - c)\mathbb{I}_{ab}/4 + c|\psi_{ab}^-\rangle\langle\psi_{ab}^-|, \quad (15)$$

where $|\psi_{ab}^-\rangle = (|0_a 1_b\rangle - |1_a 0_b\rangle)/\sqrt{2}$. These two subsystems are then let to interact locally with two independent environments in the vacuum state $|0_{E_s}\rangle$, where $s = a, b$. So, the initial state of the whole system is

$$\rho_{abE_a E_b} = \rho_{ab}^w \otimes |0_{E_a}\rangle\langle 0_{E_a}| \otimes |0_{E_b}\rangle\langle 0_{E_b}|. \quad (16)$$

For this system, it was shown in Ref. [57] that, in contrast to dissipative interactions, the decoherent dynamics of the two qubits under phase-damping or Pauli channels does not generate entanglement between the systems and its respective environments or between the two environments. The bipartite quantum discord created in such a dynamics was then indicated as the mechanism for the leakage of quantum information out of the systems. Moreover, the initial quantum correlation between the two qubits was shown to be not transferred to the environments, seeming to evaporate. Here we extend these results by studying the generation of genuine multipartite system-environment correlations. We show that the quantum correlations initially shared between the qubits do not disappear, but are transformed into genuine multipartite correlations between systems and environments. Furthermore, these correlations present an interesting dynamics with sudden changes in behavior. For more results related to the sudden-change phenomenon see Refs. [58–62]. Further works considering the dynamics of system-environment correlations can be found in Refs. [63–68].

A. Amplitude-damping channels

We begin by studying the situation in which the two qubits, a and b , evolve under the influence of local-independent amplitude-damping channels. The Kraus' operators for a dissipative reservoir at zero temperature are $K_0 = |0_s\rangle\langle 0_s| + \sqrt{1 - p_s}|1_s\rangle\langle 1_s|$ and $K_1 = \sqrt{p_s}|0_s\rangle\langle 1_s|$, where p_s is a parametrization of time for the subsystem s , with $p = 0$ corresponding to $t = 0$ and $p = 1$ being equivalent to $t \rightarrow \infty$ [69]. Throughout this article we consider identical environments and consequently $p_a = p_b := p$. Thus, by using the Kraus operators shown above, we obtain the dynamical map for the system-environment evolution:

$$\begin{aligned} U_{sE_s}|0_s 0_{E_s}\rangle &= |0_s 0_{E_s}\rangle, \\ U_{sE_s}|1_s 0_{E_s}\rangle &= \sqrt{1 - p_s}|1_s 0_{E_s}\rangle + \sqrt{p_s}|0_s 1_{E_s}\rangle. \end{aligned} \quad (17)$$

Utilizing this map, we find the evolved state for the whole system:

$$\rho_{abE_a E_b}^{ad}(p) = (1 - c)\iota_{ad}(p) + c|\Upsilon_{ad}(p)\rangle\langle\Upsilon_{ad}(p)|, \quad (18)$$

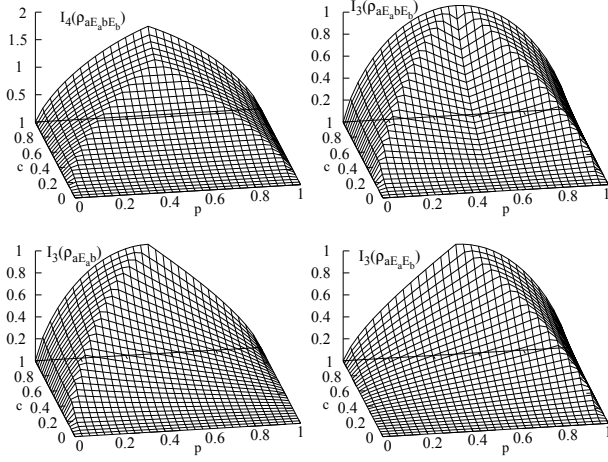


Figure 1: Genuine 4- and 3-partite total correlation of the whole system $aE_a bE_b$ and genuine 3-partite correlation of abE_a and $aE_a E_b$ for local-dissipative environments. We see that, in general, the multipartite correlations increase with p up to a certain instant of time from which they start to diminish going to zero in the asymptotic limit $p = 1$.

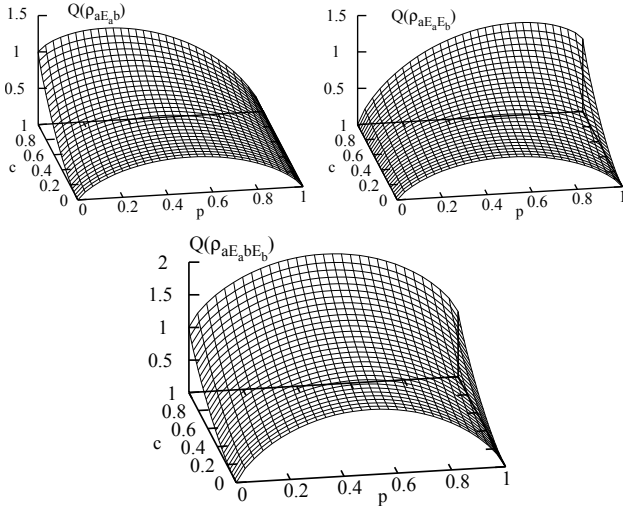


Figure 2: Multipartite system-environment quantum correlations for local amplitude-damping channels. For $p = 1$ the systems state is $|0_s\rangle$ and its initial correlations were altogether transferred to the environments.

with $\iota_{ad}(p)$ and $|\Upsilon_{ad}(p)\rangle$ given in Appendix A. From $\rho_{abE_a E_b}^{ad}(p)$ we can access the state of any partition of the system and calculate numerically its entropy and correlations. Here we will concentrate on multipartite correlations. A detailed analysis of the bipartite correlations can be found in Ref. [57]. We observe that, due to the assumed symmetries of the systems and environments, the following partitions are equivalent in what refer to correlations: $aE_a = bE_b$, $aE_b = bE_a$, $abE_a = abE_b$, and $aE_a E_b = bE_a E_b$. Considering these symmetries we have, from the definition of genuine n -partite total correlation

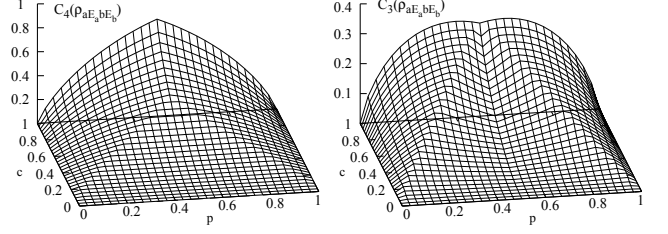


Figure 3: Genuine multipartite system-environment classical correlation for local amplitude-damping channels. The generic behavior of these correlations is similar to that of genuine multipartite total correlation.

introduced in Section II, that

$$I_4(\rho_{aE_a bE_b}) = \min_{(c_1, c_2)} (S(\rho_{c_1}) + S(\rho_{c_2}) - S(\rho_{aE_a bE_b}^{ad})), \quad (19)$$

with the following possible bi-partitions of the system: $(c_1, c_2) = (a, E_a bE_b), (E_a, abE_b), (ab, E_a E_b), (aE_a, bE_b), (aE_b, bE_a)$. The 3-partite genuine total correlations of the whole system is given by

$$I_3(\rho_{aE_a bE_b}^{ad}) = \max(I_3(\rho_{aE_a b}^{ad}), I_3(\rho_{aE_a E_b}^{ad})) \quad (20)$$

with analogous bi-partitions used to compute the genuine 3-partite correlation of the 3-partite states. All genuine total correlations discussed above are shown in Fig. 1. We see that, in general, multipartite correlations are generated during the dissipative interaction between the qubits and its respective reservoirs up to a certain instant of time from which such correlations suddenly begin to decrease going to zero in the asymptotic time $p = 1$. We also calculated the 3- and 4-partite quantum correlations and genuine classical correlations of the system, which are presented in Figs. 2 and 3, respectively. As expected, the asymptotic behavior of the genuine 3- and 4-partite classical correlations is similar to that of genuine multipartite total correlation. We observe that the quantum correlation remaining at $p = 1$ is due solely to the environments, once the systems state in this limit is $|0_s\rangle$. It is worth mentioning that, although explicit parametrizations for states and unitary operators of systems with dimension greater than two are possible in principle [70], all the important aspects we want to emphasize here can be addressed without computing the genuine multipartite quantum correlations. We leave related issues for future investigations.

B. Phase-damping channels

Let us consider the dynamics of two qubits under local phase-damping channels. This kind of noise environment causes loss of phase relations in the system without exchange of energy. The Kraus' operators for these channels are given by: $K_0 = |0_s\rangle\langle 0_s| + \sqrt{1-p}|1_s\rangle\langle 1_s|$ and $K_1 = \sqrt{p}|1_s\rangle\langle 1_s|$. Thus, the following map for the system-environment evolution is obtained

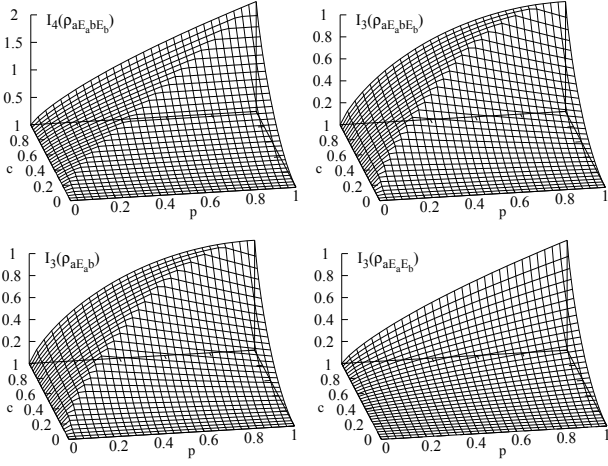


Figure 4: Genuine 4- and 3-partite total correlation of the whole system $aE_a bE_b$ and genuine 3-partite correlation of abE_a and $aE_a E_b$ for local phase-damping channels. In this case, the genuine multipartite correlations generally increase with time and, in contrast to amplitude-damping channels, there exists a considerable amount of genuine n -partite total correlation left over in the asymptotic limit $p = 1$, depending on the purity of the initial two-qubit state.

$$\begin{aligned} U_{sE}|0_s 0_{E_s}\rangle &= |0_s 0_{E_s}\rangle, \\ U_{sE}|1_s 0_{E_s}\rangle &= |1_s\rangle \otimes \left(\sqrt{1-p}|0_{E_s}\rangle + \sqrt{p}|1_{E_s}\rangle \right). \end{aligned} \quad (21)$$

In a correspondent manner as we did for amplitude-damping channels, we use the map shown in the last equation to compute the global evolved state and then calculate its correlations. In this case, $\rho_{aE_a bE_b}^{pd}(p)$ is given as in Eq. (18) but with $\iota_{pd}(p)$ and $|\Upsilon_{pd}(p)\rangle$ presented in Appendix B. The genuine total correlations generated in the evolution under local phase environments are shown in Fig. 4. In sharp contrast to the case of amplitude-damping channels, we see that the dynamics induced by phase environments does generate genuine multipartite correlations in the asymptotic limit $p = 1$. In this limit, the whole system will be correlated in a degree that is proportional to the purity of the two-qubit initial state. We also calculated the multipartite quantum correlations and the genuine 3- and 4-partite classical correlations, which are presented in Figs. 5 and 6, respectively. It is interesting that the genuine 3-partite classical and total correlations range from zero to one in this kind of environment. This indicates that the system presents no or small genuine 3-partite quantum correlation. As can be seen in Figs. 4-6, this situation changes in the case of 4-partite correlations. In fact, we observe in Fig. 5 that tripartite quantum correlations are created during the system's evolution but disappear in the asymptotic limit while there are 4-partite quantum correlation remnant at $p = 1$.

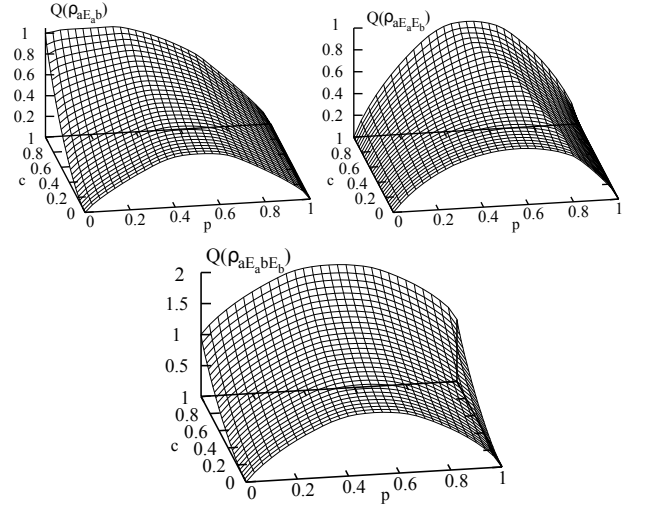


Figure 5: Dynamics of 3- and 4-partite system-environment quantum correlations for local phase-damping channels. Tripartite quantum correlations are generated during the system's evolution but decay to zero at $p = 1$ while there still exists multipartite quantum correlation involving the whole system in this asymptotic limit.

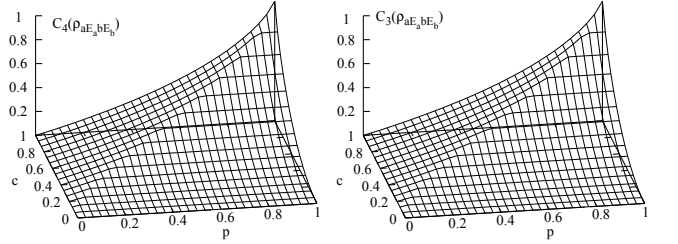


Figure 6: Genuine 3- and 4-partite system-environment classical correlation for local phase-damping channels. By comparing these results with the genuine total correlations in Fig. 4, we see that the genuine 3-partite classical and total correlations have the same range and behavior. Therefore there is no or feeble quantumness in the genuine tripartite correlations of the system. However, as can be observed in Fig. 5, there exists 4-partite quantum correlation that was created during the system-environment interaction.

IV. CONCLUDING REMARKS

Summing up, we defined measures for genuine n - and k -partite total, quantum, and classical correlations of a n -partite state. Using these correlation measures, we showed that, in contrast to amplitude-damping channels, for which the initial correlations between the qubits are simply transferred to the environments, phase noise channels turn such bipartite correlations into genuine multipartite system-environment total, quantum, and classical correlations.

In order to get a better grasp of these results, let's have a look at the kind of state generated during the evolution of the system under different kinds of noise environment. It is straightforward to verify that if the system initial

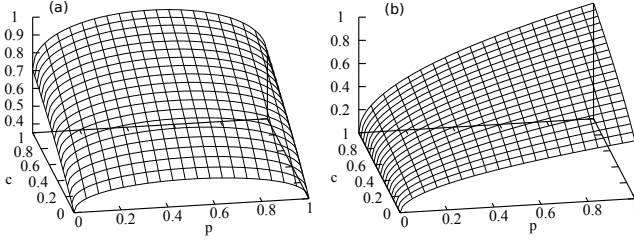


Figure 7: Dependence of the fidelities (a) $F(|\psi_W\rangle, \rho_{aE_a b E_b}^{ad}(p))$ and (b) $F(|\psi_{GHZ}\rangle, \rho_{aE_a b E_b}^{pd}(p))$ with p and c . See the text for details.

state is $|\psi_{ab}^-\rangle$ (i.e., if $c = 1$), the whole system state for amplitude-damping channels and $p = 1/2$ is

$$\begin{aligned} |\Upsilon_{ad}(1/2)\rangle &= (|0001\rangle + |0010\rangle - |0100\rangle - |1000\rangle)/2 \\ &:= |\psi_W\rangle, \end{aligned} \quad (22)$$

which is equivalent, modulo local rotations, to the well known four-qubit W state [71]. For phase noise environments and $p = 1$, it follows that

$$\begin{aligned} |\Upsilon_{pd}(1)\rangle &= (|0011\rangle - |1100\rangle)/\sqrt{2} \\ &:= |\psi_{GHZ}\rangle, \end{aligned} \quad (23)$$

which is, also modulo local unitaries, equivalent to a GHZ state (see Section I). For any c and p , we obtain the following expressions for the fidelity,

$$F(|\psi\rangle, \rho) := \sqrt{\langle\psi|\rho|\psi\rangle}, \quad (24)$$

between these states and its corresponding evolved density operators:

$$F(|\psi_W\rangle, \rho_{aE_a b E_b}^{ad}(p)) = \sqrt{\frac{(1+3c)(1+2\sqrt{p(1-p)})}{8}} \quad (25)$$

and

$$F(|\psi_{GHZ}\rangle, \rho_{aE_a b E_b}^{pd}(p)) = \sqrt{(1+3c)p}/2. \quad (26)$$

These fidelities are shown graphically in Fig. 7. We observe that the mutual information is invariant under local unitary transformations and, as one can easily verify, the genuine total correlations for the W and GHZ states are given by: $I_4(|W\rangle) = I_4(|GHZ\rangle) = 2$, $I_3(|W\rangle) = 0.81$, and $I_3(|GHZ\rangle) = 1$. Moreover, if we trace out a subsystem of a GHZ state, the obtained tripartite density operator has zero quantum correlations. The same action in a W state produces a 3-partite system possessing quantumness in its correlations. Thus, the values of the fidelities (shown in Fig. 7) and the structure of W and GHZ states with respect to its correlations help us to partially explain the general behavior of the correlations that we presented and discussed in the last section. In reality, both kinds of dynamics generated genuine multipartite correlations. The main difference is that for the

amplitude-damping channel these correlations are null for $p = 1$ while for phase environments they generally reach its maximum value in this limit.

Besides the actual calculation of the genuine multipartite quantum correlations, other interesting topic for future research is considering the composition of both phase and amplitude channels, which is the most common situation in nature [50, 59]. In this case, the global state generated during the evolution under local environments will be a mixture involving W and GHZ components. The extension of these results for global environments is also worth pursuing [72].

It is important to mention that the first experiment with complete tomography of the environment's state has been successfully performed recently using an optical system [73]. Therefore, all the theoretical results presented here can be verified experimentally with current technology.

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Appendix A: Evolved State for Local Amplitude-Damping Channels

For this channels, the global evolved state is given by Eq. (18) with

$$\begin{aligned} 4\iota_{ad}(p) &= |0\rangle\langle 0| + (1-p)(|2\rangle\langle 2| + |8\rangle\langle 8|) \\ &\quad + p^2|5\rangle\langle 5| + (1-p)^2|10\rangle\langle 10| \\ &\quad + p(1-p)(|6\rangle\langle 6| + |9\rangle\langle 9|) + p(|4\rangle\langle 4| + |1\rangle\langle 1|) \\ &\quad + \sqrt{p(1-p)}(|4\rangle\langle 8| + |1\rangle\langle 2| + \text{h.c.}) \\ &\quad + \sqrt{p(1-p)^3}((|6\rangle + |9\rangle)\langle 10| + \text{h.c.}) \\ &\quad + p(1-p)(|5\rangle\langle 10| + |6\rangle\langle 9| + \text{h.c.}) \\ &\quad + \sqrt{p^3(1-p)}(|5\rangle\langle 6| + |9\rangle\langle 1| + \text{h.c.}) \end{aligned} \quad (A1)$$

and

$$|\Upsilon_{ad}(p)\rangle = \frac{1}{\sqrt{2}} \left(\sqrt{1-p}(|2\rangle - |8\rangle) + \sqrt{p}(|1\rangle - |4\rangle) \right). \quad (A2)$$

Above, and in the next appendix, h.c. refers to the Hermitian conjugate and we use the decimal representation for the indexes of the computational bases states.

Appendix B: Evolved State for Local Phase-Damping Channels

For phase environments, the global evolved state is given as in Eq. (18) but with

$$\begin{aligned} 4\iota_{pd}(p) = & |0\rangle\langle 0| + (1-p)(|2\rangle\langle 2| + |8\rangle\langle 8|) \\ & + p(|3\rangle\langle 3| + |12\rangle\langle 12|) + (1-p)^2|10\rangle\langle 10| \\ & + p(1-p)(|11\rangle\langle 11| + |14\rangle\langle 14|) + p^2|15\rangle\langle 15| \\ & + \sqrt{p(1-p)}(|2\rangle\langle 3| + |8\rangle\langle 12| + \text{h.c.}) \end{aligned}$$

and

$$|\Upsilon_{pd}(p)\rangle = \frac{1}{\sqrt{2}} \left(\sqrt{1-p}(|2\rangle - |8\rangle) + \sqrt{p}(|3\rangle - |12\rangle) \right). \quad (\text{B2})$$

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- [1] N. D. Mermin, Rev. Mod. Phys. **65**, 803 (1993).
 - [2] A. Aspect, arXiv:quant-ph/0402001v1.
 - [3] R. Horodecki *et al.*, Rev. Mod. Phys. **81**, 865 (2009).
 - [4] O. Ghne and G. Tth, Phys. Rep. **474**, 1 (2009).
 - [5] L. C. Cleri, J. Maziero, and R. M. Serra, Int. J. Quant. Inf. **9**, 1837 (2011).
 - [6] K. Modi *et al.*, arXiv:1112.6238v1.
 - [7] B. Groisman, S. Popescu, and A. Winter, Phys. Rev. A **72**, 032317 (2005).
 - [8] L. Henderson and V. Vedral, J. Phys. A: Math. Gen. **34**, 6899 (2001).
 - [9] H. Ollivier and W. H. Zurek, Phys. Rev. Lett. **88**, 017901 (2001).
 - [10] M. Horodecki *et al.*, Phys. Rev. A **71**, 062307 (2005).
 - [11] M. Piani, P. Horodecki, and R. Horodecki, Phys. Rev. Lett. **100**, 090502 (2008).
 - [12] D. Cavalcanti *et al.*, Phys. Rev. A **83**, 032324 (2011).
 - [13] V. Madhok and A. Datta, Phys. Rev. A **83**, 032323 (2011).
 - [14] A. Ferraro *et al.*, Phys. Rev. A **81**, 052318 (2010).
 - [15] W. H. Zurek, Phys. Rev. A **67**, 012320 (2003).
 - [16] G. Adesso and A. Datta, Phys. Rev. Lett. **105**, 030501 (2010).
 - [17] P. Giorda and M. G. A. Paris, Phys. Rev. Lett. **105**, 020503 (2010).
 - [18] M. F. Cornelio, M. C. de Oliveira, and F. F. Fanchini, Phys. Rev. Lett. **107**, 020502 (2011).
 - [19] A. Streltsov, H. Kampermann, and D. Bruß, Phys. Rev. Lett. **106**, 160401 (2011).
 - [20] J. Maziero and R. M. Serra, Int. J. Quant. Inf. **10**, 1250028 (2012).
 - [21] G. H. Aguilar *et al.*, Phys. Rev. Lett. **108**, 063601 (2012).
 - [22] D. Kaszlikowski *et al.*, Phys. Rev. Lett. **101**, 070502 (2008).
 - [23] Z. Walczak, Phys. Lett. A **374**, 3999 (2010).
 - [24] A. Grudka *et al.*, arXiv:0802.1633v1.
 - [25] A. SaiToh, R. Rahimi, and M. Nakahara, Int. J. Quant. Inf. **6**, 787 (2008).
 - [26] C. C. Rulli and M. S. Sarandy, Phys. Rev. A **84**, 042109 (2011).
 - [27] I. Chakrabarty, P. Agrawal, and A. K. Pati, Eur. Phys. J. D **65**, 605 (2011).
 - [28] M. Okrasa and Z. Walczak, EPL **96**, 60003 (2011).
 - [29] P. Parashar and S. Rana, Phys. Rev. A **83**, 032301 (2011).
 - [30] S. Campbell, L. Mazzola, and M. Paternostro, Int. J. Quant. Inf. **9**, 1685 (2011).
 - [31] G. L. Giorgi and T. Busch, arXiv:1206.1726v1.
 - [32] K. Byczuk *et al.*, Phys. Rev. Lett. **108**, 087004 (2012).
 - [33] M. Ramzan, arXiv:1205.3133v1.
 - [34] Z. Zhang, B. Ye, and S.-M. Fei, arXiv:1206.0221v1.
 - [35] Y.-K. Bai *et al.*, arXiv:1206.2096v1.
 - [36] A. L. Grimsmo, S. Parkins, B.-S. K. Skagerstam, arXiv:1205.5969v1.
 - [37] M. Mahdian, R. Yousefjani, and S. Salimi, Eur. Phys. J. D **66**, 133 (2012).
 - [38] J. Xu, arXiv:1204.5868v1.
 - [39] B. Li, L. C. Kwek, and H. Fan, arXiv:1205.6016v1.
 - [40] A. Saguia *et al.*, Phys. Rev. A **84**, 042123 (2011).
 - [41] K. Modi *et al.*, Phys. Rev. Lett. **104**, 080501 (2010).
 - [42] G. L. Giorgi *et al.*, Phys. Rev. Lett. **107**, 190501 (2011).
 - [43] C. H. Bennett *et al.*, Phys. Rev. A **83**, 012312 (2011).
 - [44] W. H. Zurek, Rev. Mod. Phys. **75**, 715 (2003).
 - [45] H. P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, Oxford, 2002).
 - [46] M. Schlosshauer, *Decoherence and the Quantum-to-Classical Transition* (Springer, Berlin, 2007).
 - [47] M. A. Nielsen and I. L. Chuang, *Quantum Information and Quantum Computation* (Cambridge University Press, Cambridge, 2000).
 - [48] J. Preskill, *Quantum Information and Computation*, <http://theory.caltech.edu/people/preskill/ph229/>.
 - [49] N. G. de Almeida, R. Napolitano, and M. H. Y. Moussa, Phys. Rev. A **62**, 033815 (2000).
 - [50] D. O. Soares-Pinto *et al.*, Phys. Rev. A **83**, 062336 (2011).
 - [51] I. L. Chuang and M. A. Nielsen, J. Mod. Opt. **44**, 2455 (1997).
 - [52] L. M. K. Vandersypen and I. L. Chuang, Rev. Mod. Phys. **76**, 1037 (2005).
 - [53] D. M. Greenberger, M. A. Horne, and A. Zeilinger, arXiv:0712.0921v1.
 - [54] D. Bouwmeester *et al.*, Phys. Rev. Lett. **82**, 1345 (1999).
 - [55] V. Vedral, Rev. Mod. Phys. **74**, 197 (2002).
 - [56] S. Sachdev, *Quantum Phase Transitions* (Cambridge University Press, New York, 2011).
 - [57] J. Maziero *et al.*, Phys. Rev. A **81**, 022116 (2010).
 - [58] J. Maziero *et al.*, Phys. Rev. A **80**, 044102 (2009).
 - [59] R. Auccaise *et al.*, Phys. Rev. Lett. **107**, 140403 (2011).
 - [60] J.-S. Xu *et al.*, Nat. Commun. **1**, 7 (2010).
 - [61] L. Mazzola, J. Piilo, and S. Maniscalco, Phys. Rev. Lett. **104**, 200401 (2010).
 - [62] M. F. Cornelio *et al.*, arXiv:1203.5068v1.

- [63] C. E. López *et al.*, Phys. Rev. Lett. **101**, 080503 (2008).
- [64] Y.-J. Zhang and Y.-J. Xia, Int. J. Quant. Inf. **7**, 949 (2009).
- [65] A. Pernice and W. T. Strunz, Phys. Rev. A **84**, 062121 (2011).
- [66] S. Luo, S. Fu, and N. Li, Phys. Rev. A **82**, 052122 (2010).
- [67] Z.-X. Man, Y.-J. Xia, and N. B. An, J. Phys. B: At. Mol. Opt. Phys. **44**, 095504 (2011).
- [68] F. F. Fanchini *et al.*, New J. Phys. **14**, 013027 (2012).
- [69] A. Salles *et al.*, Phys. Rev. A **78**, 022322 (2008).
- [70] E. Brünig *et al.*, J. Mod. Opt. **59**, 1 (2012).
- [71] W. Dür, G. Vidal, and J. I. Cirac, Phys. Rev. A **62**, 062314 (2000).
- [72] D. O. Soares-Pinto *et al.*, Phys. Rev. A **81**, 062118 (2010).
- [73] O. Jiménez Farías *et al.*, Submitted to Phys. Rev. Lett. (2012).